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BOOK REVIEWS

Book Review Editor: Walter Van Assche

Books

George G. Lorentz, Manfred V. Golitschek, and Yuly Makovoz, *Constructive Approximation: Advanced Problems*, Grundlehren der mathematischen Wissenschaften **304**, Springer-Verlag, Berlin/Heidelberg, 1996, xii + 649 pp.

Approximation theory is a diverse area, and this book clearly shows the width and depth of the subject. As a sequel to the monograph of R. A. DeVore's and G. G. Lorentz's "Constructive Approximation" (Vol. 303 of the same series, see the book review in *J. Approx. Theory* **78** (1994), 466–467), which presented a clear and systematic introduction to the approximation of functions of one real variable, this second volume, with partially different authors, treats a large number of additional questions in more detail. The range of topics presented is amazing and shows the rich diversity of the theory. Everyone working in approximation theory will be able to find something close to his area of interest. As a consequence of the diverse nature of the present volume, it is less coherent than its predecessor. Most of the 17 chapters can be read independently from one another.

Chapters 1–4 deal with polynomial approximations. Some of the many topics are the distribution of zeros and alternation points of best approximations, constrained polynomial approximation, approximation by incomplete polynomials, and approximation by weighted polynomials with varying weights. One of the highlights is a self-contained proof of a result of Lubinsky and Saff on weighted approximation by Freud weights.

Chapters 5 and 6 are about wavelets and splines. The chapter on wavelets presents the basic idea of multiresolution analysis and includes a recent construction, due to R. A. Lorentz and Sahakian, of orthonormal Schauder bases with trigonometric polynomials of low degree. The chapter on splines is a continuation of the discussion in the first volume. Splines of best approximation, periodic splines and the Schoenberg spline operator are treated.

Chapters 7–10 cover rational approximation. The authors discuss rational approximation for individual functions like e^x on $[-1, 1]$ and Popov's results for rational approximation of function classes. A separate chapter is devoted to Stahl's remarkable theorem on the error of best uniform rational approximation of $|x|$ on $[-1, 1]$. The short chapter on Padé approximation includes the Nuttall–Pommerenke theorem on convergence in capacity. This chapter is somewhat outside the main scope of the book, since it deals essentially with complex approximation. The same can be said about Chapter 10, which was prepared by Pekarskii and is based on his work on Hardy space methods for the error in best approximation.

The discussion of Müntz polynomials in Chapter 11 includes Jackson theorems and Markov-type inequalities. Some topics in nonlinear approximation are treated in Chapter 12. One finds Rice's theory of varisolvent families and abstract approximation in Banach spaces.

Chapters 13–15 discuss widths and entropies of classes of functions. Important results that are treated are the calculation of the n-widths of Sobolev classes, including recent results of Buslaev and Tikhomirov, and the asymptotics for n-widths of Lipschitz classes due to Kashin, Maiorov, Höllig, and others. Based on results in arbitrary Banach spaces, the metric entropies of unit balls in Lipschitz spaces and spaces of analytic functions are estimated.

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Chapters 16 and 17 cover convergence of sequences of operators, i.e., Korovkin-type theorems, and the representation of functions of several variables by superpositions of functions of fewer variables. Finally, there are four appendices, a large bibliography, and author and subject indexes.

This book clearly is of interest to anyone working in approximation theory.

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Gilbert G. Walter, *Wavelets and Other Orthogonal Systems with Applications*, CRC Press, Boca Raton, FL, 1994, x + 248 pp.

There has been an almost bewildering explosion in wavelet theory since the beginning of the 1990s. To the unwaved, such as the reviewer, the very large number of review articles and books that have appeared has left confusion as to where to start to learn this important subject. More specifically, as an academic with limited time, one wants to be able to pick up a book, to read it, and to incorporate parts of it into a course on approximation theory or harmonic analysis, or even to present a whole graduate course on the subject.

The book under review must surely be one of the best candidates for this purpose. There are many very readable accounts of the theory, starting from different viewpoints, such as applications to signal processing or an account of the analysis. Many start with an account of the relation to Fourier transforms and series, or to general orthogonal series. In this book, the author has integrated the theory of general orthogonal systems, and especially orthogonal polynomials with wavelet theory, to a far greater degree. In that sense, it is less specialized than most of the other books and easier for those (such as the reviewer) that come from a background of orthogonal polynomials or classical analysis.

The author notes that his target audience consists of engineering and mathematics graduate students, and that large parts of the book have been tested on a mixed class of engineering and graduate students. He also notes that the book could be used for courses on special functions or signal processing. Certainly the attractive print and style of presentation, the numerous examples given, and the introductions given to topics such as tempered distributions, sampling theorems, and statistical applications make even individual chapters attractive for selection for diverse courses. There is also a "nice light feel" to individual sections: ideas are presented without clobbering the reader with too much technical detail.

Chapter 1 contains an introduction to orthogonal functions and series, with specific attention to the trigonometric/Fourier system, and to the Shannon and Haar systems as precursors of wavelets. Chapter 2 contains the basic theory of tempered distributions. In Chapter 3, the author begins the discussion of wavelets, with multiresolution analysis, and mother wavelets, followed by several examples. Mallat's decomposition and reconstruction algorithm is presented, as well as an application to filters.

In Chapter 4, some of the classical convergence theorems for Fourier series are presented, together with less familiar ones for Fourier series of periodic distributions. Abel summability and Fejér means are also briefly discussed. In Chapter 5, the relationship between wavelets and distributions is investigated, involving topics such as wavelets based on distributions or multiresolution analysis of distributions.

In Chapter 6, classical orthogonal polynomials (Jacobi, Hermite, and others) are discussed, while orthogonal systems generated by Sturm–Liouville equations and the Walsh/Rademacher systems are presented in Chapter 7. The latter also treats orthogonal and biorthogonal wavelets. In Chapter 8, general so-called "delta sequences" are introduced, the classic examples of course being suitably scaled convolution kernels. This is applied to pointwise